

# Corso di Teoria dei Segnali

## a.a. 2010-2011

Esercitazione n. 5 – Potenza, PSD e Autocorrelazione

## POTENZA DEI SEGNALE

ESEMPLO 27)

SIANO  $v(t)$  E  $i(t)$  I SEGNALE DI TENSIONE E CORRENTE CHE  
ALL'INTERNO DI UN CIRCUITO, ALLORASI I POTERI:

$$v(t) = V \cos(\omega t) \quad \text{e} \quad i(t) = I \cos(\omega t) \quad . \quad \text{CALCOLARE:}$$

$$a) p(t) = v(t) i(t) = VI \cos^2(\omega t) = \boxed{\frac{1}{2} VI (1 + \cos 2\omega t)} \quad \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

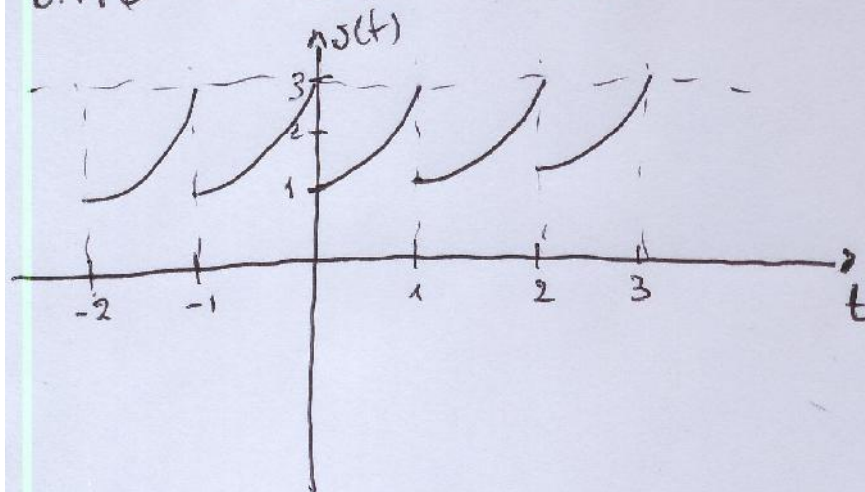
POTENZA  
ISTANTANEA

$$b) \text{ potenza media: } \langle p(t) \rangle \quad \text{con} \quad \langle f(t) \rangle = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t) dt \Rightarrow$$

$$\Rightarrow \langle p(t) \rangle = \left\langle \frac{1}{2} VI (1 + \cos 2\omega t) \right\rangle = \frac{VI}{2T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 2\omega t) dt =$$
$$= \frac{VI}{2T_0} \int_{-T_0/2}^{T_0/2} dt + \frac{VI}{2T_0} \int_{-T_0/2}^{T_0/2} \cos 2\omega t dt = \frac{VI}{2T_0} \cdot T_0 + \frac{VI}{2T_0} \cdot 0 = \frac{VI}{2}$$

### ESEMPIO 28)

DATO IL SEGNALE  $v(t)$  PERIODICO CON  $v(t) = e^t$  per  $0 < t < 1$ , CALCOLARE:



a) VALORE MEDIO ( $V_{DC}$ ):

$$\langle v(t) \rangle = \frac{1}{T_0} \int_0^{T_0} v(t) dt = \int_0^1 e^t dt = e^1 - e^0 = 2,718 - 1 = 1,714 \text{ V}$$

b) VALORE RMS (VALORE Q. MEDIO, V. EFFICACE)

$$V_{RMS}^2 = \langle v^2(t) \rangle = \int_0^1 (e^t)^2 dt = \frac{1}{2} (e^2 - e^0) = \frac{1}{2} (7,389 - 1) = 3,194 \Rightarrow V_{RMS} = 1,79 \text{ V}$$

c) SEGNALE APPLICATO AI CAPI DI CARICO RESISTIVO  $R = 600 \Omega$ , QUANTO VALS LA POT. DISSIPATA SUL CARICO?

$$P = V_{RMS}^2 / R = (1,79)^2 / 600 = 5,32 \times 10^{-3} \text{ W}$$



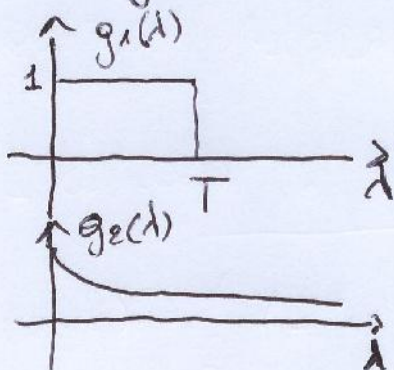
# ESEMPIO 29)

DATI 2 SEGNALE  $g_1(t)$  E  $g_2(t)$ , CALCOLARE

$$g_3(t) = g_1(t) \otimes g_2(t)$$

$$g_1(t) = \Pi\left(\frac{t - \frac{1}{2}T}{T}\right)$$

$$g_2(t) = e^{-t/T} u(t)$$



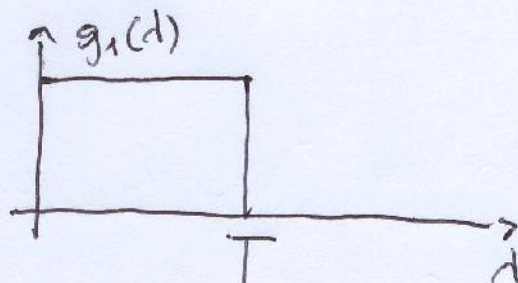
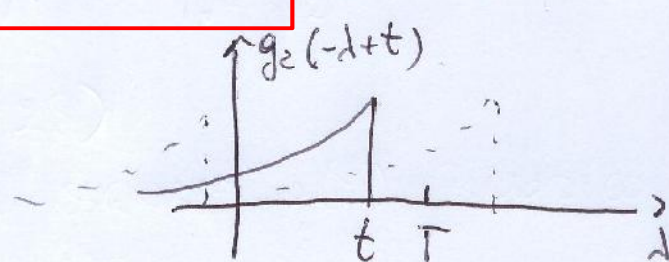
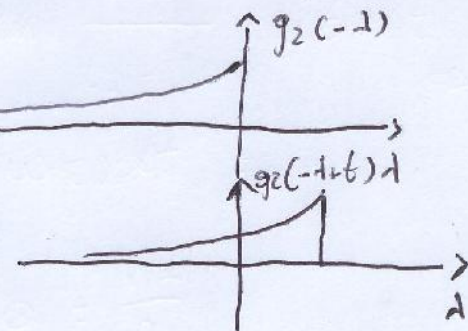
$$g_3(t) = \int_{-\infty}^{+\infty} g_1(\lambda) g_2(t-\lambda) d\lambda$$

PER OTTENERE LA FUNZIONE INTEGRANDA PROCEDIAMO PER VIA GRAFICA:

1) INVERSIONE SUCCESSO TEMPORALE DI  $g_2 \rightarrow g_2(-\lambda)$

2) SPOSTAMENTO DI  $t$  UNITÀ DI TEMPO  $g_2(-\lambda) \rightarrow g_2(-(\lambda-t))$

3) MOLTIPLICAZIONE CON  $g_1$



L'INTEGRAL È NULLO PER  $t < 0$  (PRODOTTO NULLO)

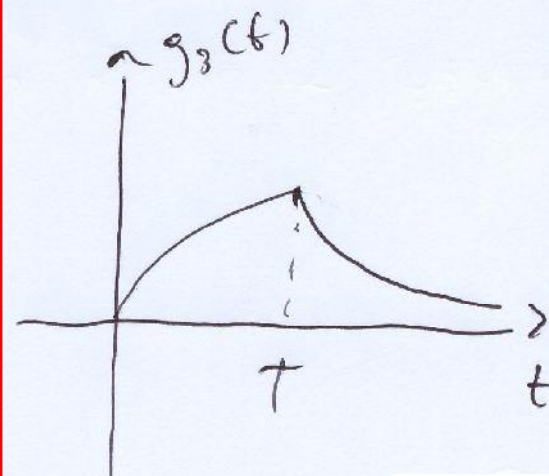
È NULLA  
 $g_2$

SE  $0 < t < T \Rightarrow g_3(t) = \int_0^t 1 \cdot e^{(\lambda-t)/T} d\lambda = T(1 - e^{-t/T})$  PRODOTTO NULLO PER  $t < \lambda < T$

SE  $t > T \Rightarrow g_3(t) = \int_0^T 1 \cdot e^{(\lambda-t)/T} d\lambda = T(e-1)e^{-t/T}$

È NULLA  
 $g_1$

$$g_3(t) = \begin{cases} 0 & t < 0 \\ T(1 - e^{-t/T}) & 0 < t < T \\ T(e-1)e^{-t/T} & t > T \end{cases}$$





PSD  $P_g(f) \triangleq \lim_{T \rightarrow \infty} \frac{|G_T(f)|^2}{T}$

$g_T(t) = g(t) \pi\left(\frac{t}{T}\right)$   
trapezoida

AUTOCOR.  $R_g(\tau) \triangleq \langle g(t) g(t+\tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t) g(t+\tau) dt$

$P_g(f) = \mathcal{F}[R_g(\tau)]$  ← metodo indiretto

ESEMPLO 30)

CALCOLARE

PSD di  $g(t) = A \sin \omega_0 t$  con rettangolo INDIRETTO:

$$R_g(\tau) = \langle g(t) g(t+\tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \sin \omega_0 t \sin \omega_0 (t+\tau) dt$$

RICORDANDO CHE:  $2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$

$\alpha = \omega_0 t$

$\beta = \omega_0 t + \omega_0 \tau$

$$R_g(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2/2 [\cos(-\omega_0 \tau) - \cos(2\omega_0 t + \omega_0 \tau)] dt \Rightarrow$$

$$\Rightarrow R_g(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} \cos \omega_0 \tau dt - \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} \cos(2\omega_0 t + \omega_0 \tau) dt =$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \frac{A^2}{2} \cos \omega_0 \tau \int_{-T/2}^{T/2} dt - \lim_{T \rightarrow \infty} \frac{1}{T} \frac{A^2}{2} \int_{-T/2}^{T/2} \underbrace{\cos(2\omega_0 t + \omega_0 \tau)}_{\text{"0"}} dt \Rightarrow$$

$$\Rightarrow R_g(\tau) = \lim_{T \rightarrow \infty} \frac{A^2}{2} \cos \omega_0 \tau = \frac{A^2}{2} \cos \omega_0 \tau$$

$$P_g(f) = \mathcal{F}\left[\frac{A^2}{2} \cos \omega_0 \tau\right] = \frac{A^2}{4} [\delta(f+f_0) + \delta(f-f_0)]$$

$$\text{POICHS} \quad \mathcal{F}[A \cos \omega_0 t] = \frac{A}{2} [\delta(f+f_0) + \delta(f-f_0)]$$

"dim. per casa"



ESSEPI0 31)

DATA  $g(t) = 5 + 12 \cos \omega_0 t$   $f_0 = 10 \text{ Hz}$ , CALCOLO DI  $R_g(\tau) = R_g(f)$ :

$$R_g(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} (5 + 12 \cos \omega_0 t) (5 + 12 \cos \omega_0 (t + \tau)) dt =$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} [25 + 60 \cos \omega_0 (t + \tau) + 6 \cos \omega_0 t + 144 \cos \omega_0 t \cos \omega_0 (t + \tau)] dt =$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \cdot 25 \int_{-T/2}^{T/2} dt + \underbrace{\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} 60 \cos \omega_0 (t + \tau) dt}_{=0} + \underbrace{\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} 6 \cos \omega_0 t dt}_{=0} +$$

$$+ \lim_{T \rightarrow \infty} \frac{1}{T} \cdot 144 \int_{-T/2}^{T/2} \cos \omega_0 t \cos \omega_0 (t + \tau) dt \Rightarrow$$

$$\Rightarrow R_g(\tau) = 25 + \lim_{T \rightarrow \infty} \left[ \frac{1}{T} \int_{-T/2}^{T/2} 72 \cos(-\omega_0 \tau) dt + \frac{1}{T} \int_{-T/2}^{T/2} 72 \cos(2\omega_0 t + \omega_0 \tau) dt \right] =$$



$$\Rightarrow P_g(z) = 25 + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} 72 \cos \omega_0 z \, dt = 25 + 72 \cos \omega_0 \tau$$

$$\alpha = \omega_0 t$$

$$\beta = \omega_0 t + \omega_0 \tau$$

$$P_g(f) = 25 \delta(f) + \frac{72}{2} [\delta(f + f_0) + \delta(f - f_0)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

P3R CASA:

CALCOLARE  $y(t) = w_1(t) \otimes w_2(t)$  CON:

$$w_1(t) = \begin{cases} 1 & \text{se } |t| < T_0 \\ 0 & \text{else} \end{cases}$$

$$w_2(t) = \begin{cases} [1 - 2|t|] & \text{se } |t| \leq \frac{T_0}{2} \\ 0 & \text{else} \end{cases}$$

**ESERCIZIO**: Dato il segnale  $g(t)$  di periodo  $T_0 = 0.5 \text{ msec}$ , con coeff. dello sviluppo in serie pari a:

$$C_m = \begin{cases} 3 & m=0 \\ m \cos(m\pi) & m \in \{+1, +2, +3\} \\ 0 & \text{altrimenti} \end{cases}$$

---	$C_{-4}$	$C_{-3}$	$C_{-2}$	$C_{-1}$	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	---
0	0	+3	-2	1	3	-1	+2	-3	0	0

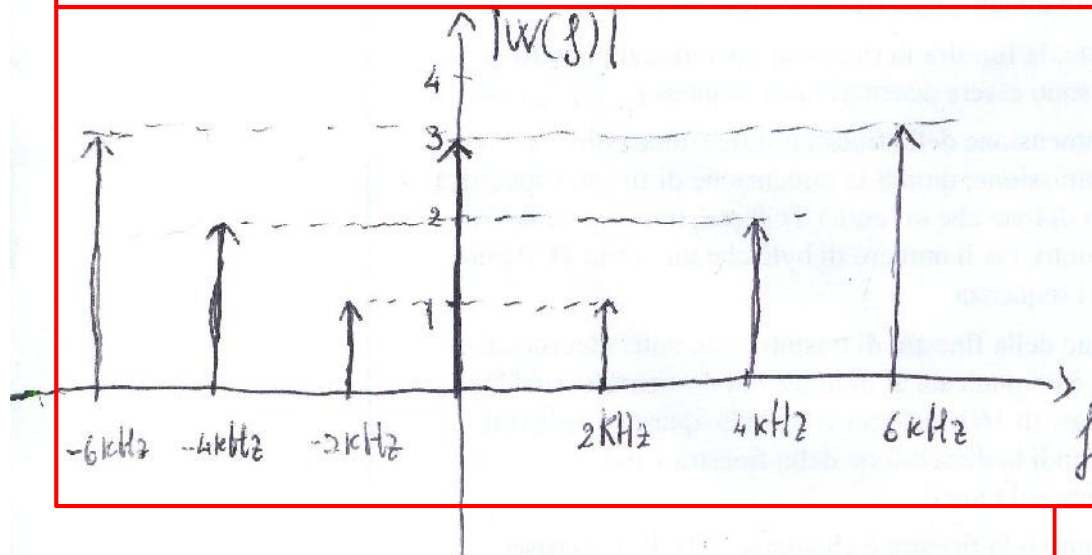
a) Scrivere  $g(t)$  in forme esponenziale:  $g(t) = \sum_{m=-\infty}^{+\infty} C_m e^{j m \omega_0 t}$

$$\begin{aligned} g(t) &= \frac{3e^{-j3 \cdot 2\pi \cdot 2000t}}{-1 \cdot e^{j2\pi \cdot 2000t}} - \frac{2e^{-j2 \cdot 2\pi \cdot 2000t}}{+2e^{j2 \cdot 2\pi \cdot 2000t}} + \frac{e^{-j2\pi \cdot 2000t}}{-3e^{j3 \cdot 2\pi \cdot 2000t}} + 3e^{j0 \cdot \pi \cdot 2000t} \\ &= -[3(e^{j \cdot 12000\pi t}) - 3(e^{-j \cdot 12000\pi t})] + 2[e^{j8000\pi t} - e^{-j8000\pi t}] + \\ &\quad -1[+e^{j4000\pi t} - e^{-j4000\pi t}] + 3 = 3 + 2j[-3 \sin(12000\pi t) + 2 \sin(8000\pi t) + \\ &\quad - \sin(4000\pi t)]. \end{aligned}$$



b) disegnare lo spettro delle ampiezze di  $g(t)$ , cioè  $|G(f)|$ .

$$|G(f)| = 3\delta(f) + 3\delta(f+3f_0) + 3\delta(f-3f_0) + 2\delta(f+2f_0) + 2\delta(f-2f_0) + \delta(f-f_0) + \delta(f+f_0)$$

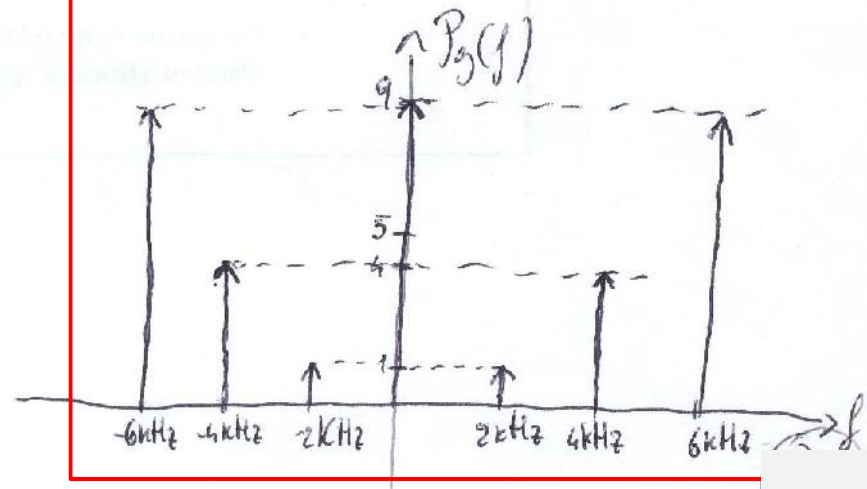


$$W(f) = \sum_{n=-\infty}^{+\infty} C_n \delta(f - n f_0):$$

c) calcolare la PSD  $P_g(f)$  di  $g(t)$ :

$$P_g(f) = \sum_{n=-\infty}^{+\infty} |C_n|^2 \delta(f - n f_0)$$

$$P_g(f) = 9\delta(f) + \delta(f-f_0) + \delta(f+f_0) + \\ + 2\delta(f-2f_0) + 2\delta(f+2f_0) + \\ + 3\delta(f-3f_0) + 3\delta(f+3f_0)$$



ESERCIZIO 32)

SIGNALS  $g(t) = 20 + 20 \sin(500t + 30^\circ)$ , CALCOLARE:

a) INTERVALLO MASSIMO TRA I CAMPIONI:

$$500 = 2\pi f_0 \Rightarrow f_0 = \frac{500}{2\pi} \approx 79.62 \text{ Hz} \Rightarrow f_{\text{min}} = 2 \cdot 79.62 \approx 160 \text{ Hz}$$

$$\Rightarrow T_{\text{min}} = 1/160 \approx 0.00628 = 6.28 \text{ ms}$$

b) QUANTI CAMPIONI DEVONO ESSERE RACCOLTI PER RIPRODURRE UN SECONDO DI SIGNALS:  $M_s = 1/6.28 \times 10^{-3} \approx 160$

PSR CASA:

CALCOLARE IL VALORE EFFICACE DI  $s(t) = A_1 \cos(\omega_1 t + \varphi_1) + A_2 \cos(\omega_2 t + \varphi_2)$  SE:

a)  $\omega_1 = \omega_2$   $\varphi_1 = \varphi_2 \Rightarrow S_{\text{eff}} = (A_1 + A_2)/\sqrt{2}$

b)  $\omega_1 = 2\omega_2$   $\varphi_1 = \varphi_2 \Rightarrow S_{\text{eff}} = (\sqrt{A_1^2 + A_2^2})/\sqrt{2}$

c)  $\omega_1 = 2\omega_2$   $\varphi_1 = \varphi_2 + \pi \Rightarrow S_{\text{eff}} = (\sqrt{A_1^2 + A_2^2})/\sqrt{2}$